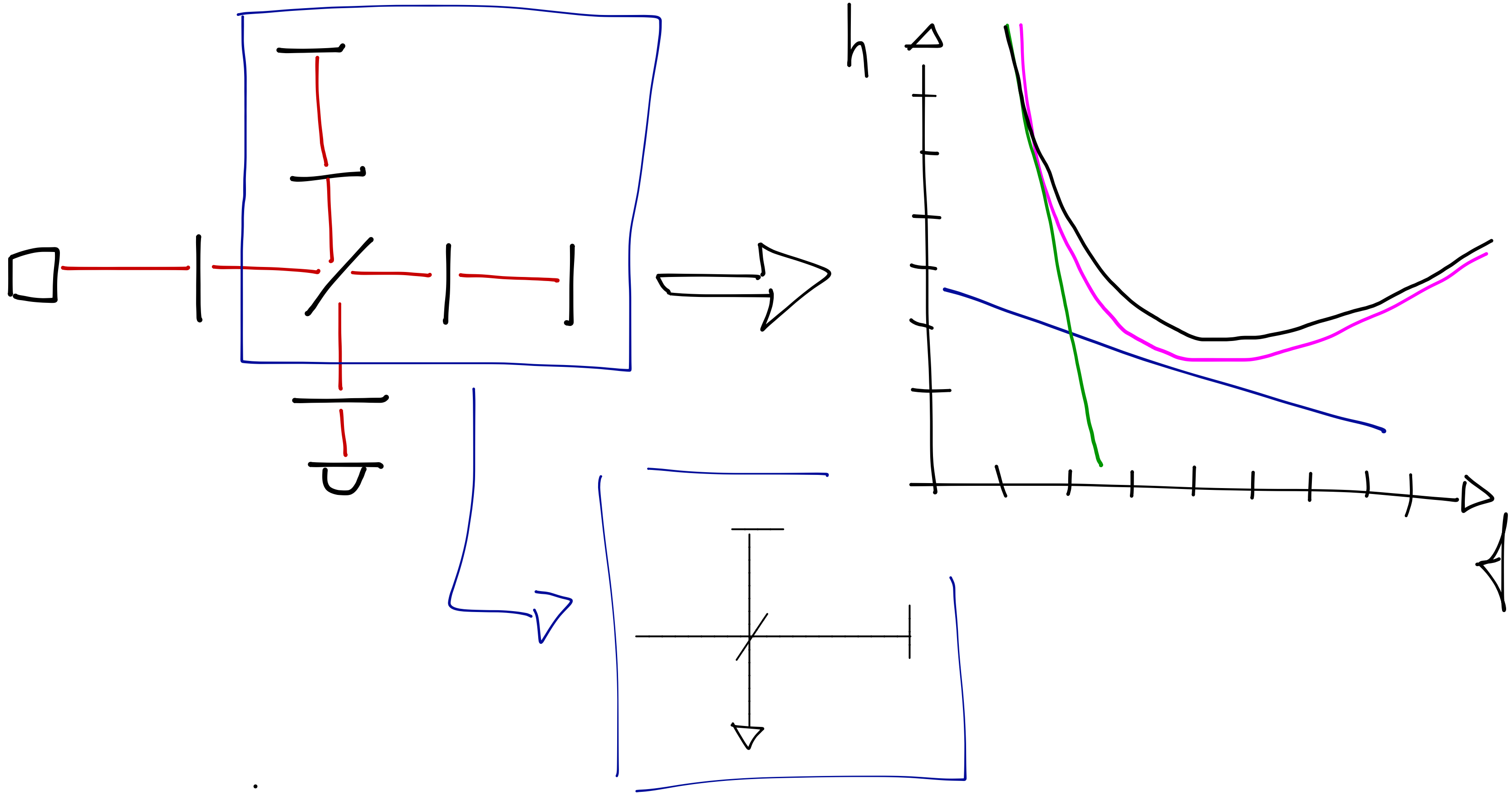
4 MICHELSON

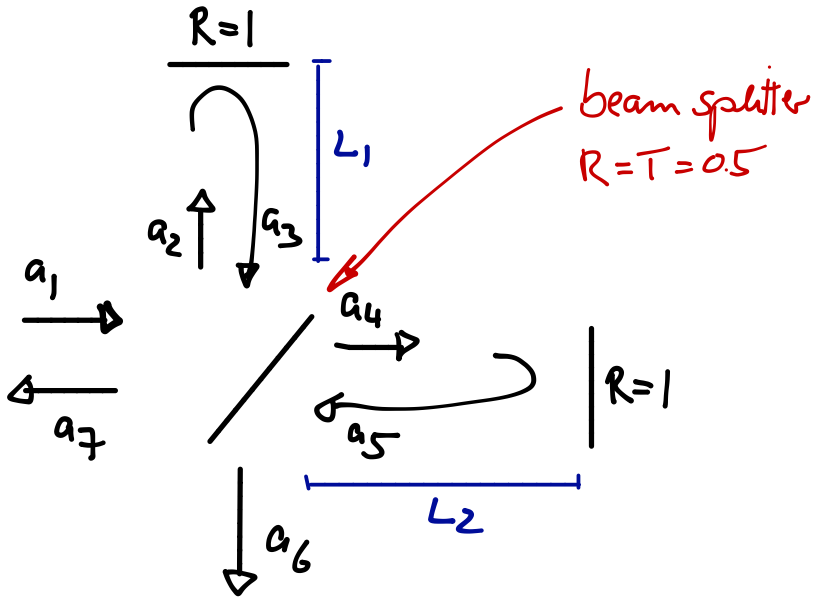


This session:

- The Michelson interferometer, the core of LIGO
- How does it work?
- Why the Michelson? And why the 'dark fringe'?

L4

Use coupling equations:



$$a_2 = \sqrt{\tfrac{1}{2}}\, a_1, \quad a_3 = a_2\, e^{-i2kL_1}$$

$$a_4 = i\sqrt{\tfrac{1}{2}}\, a_1, \quad a_5 = a_4\, e^{-i2kL_2}$$

$$a_6 = i\sqrt{\tfrac{1}{2}}\, a_3 + \sqrt{\tfrac{1}{2}}\, a_5 = i\,\tfrac{1}{2}\, e^{-i2kL_1} a_1 + i\,\tfrac{1}{2}\, e^{-i2kL_2} a_1 = i\,\tfrac{1}{2}\, a_1 \left(e^{-i2kL_1} + e^{-i2kL_2}\right)$$

$$a_7 = \sqrt{\tfrac{1}{2}}\, a_3 + i\sqrt{\tfrac{1}{2}}\, a_5$$

We can simplify using

$$\bar{L} = \tfrac{1}{2}(L_1 + L_2)$$

$$\Delta L = \tfrac{1}{2}(L_1 - L_2)$$

$$\rightarrow \bar{L} + \Delta L = L_1$$

$$\bar{L} - \Delta L = L_2$$

then

$$a_6 = a_1\, \tfrac{i}{2}\, e^{-i2\pi k\bar{L}} \underbrace{\left(e^{-i2k\Delta L} + e^{i2k\Delta L}\right)}_{=2\cos(2k\Delta L)}$$

$$= a_1\, i\, e^{-i2\pi k\bar{L}} \cos(2k\Delta L)$$

ΔL changes amplitude

$\bar{L}$ changes phase

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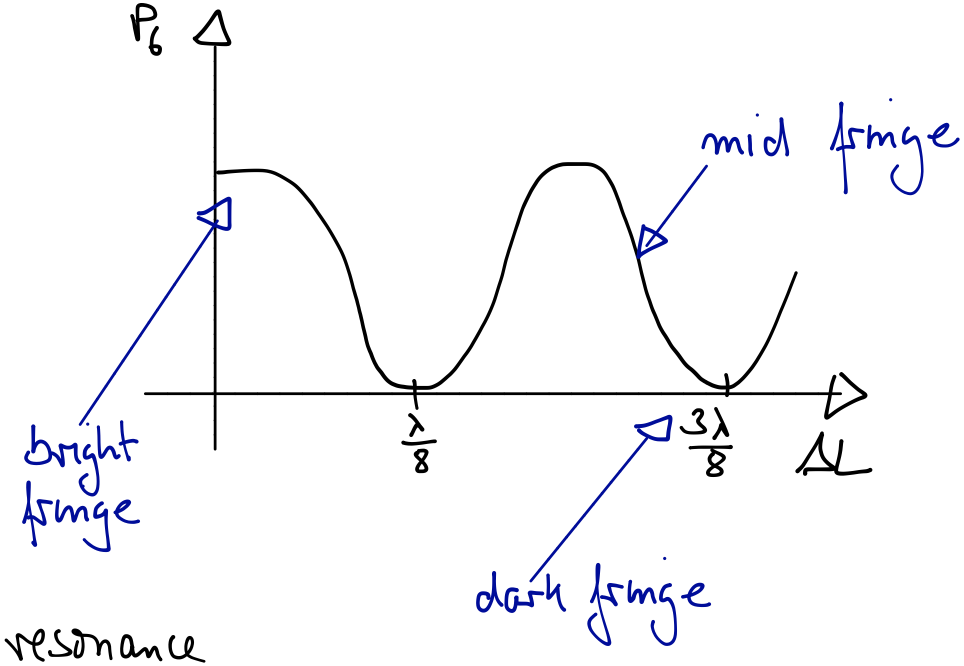
Power in output

$$P_6 = P_1 \cdot \cos^2(2k\Delta L)$$

Minimum at $2k\Delta L = (2N+1)\frac{\pi}{2}$

$\Rightarrow \Delta L = (2N+1)\frac{\lambda}{8}$



Operating points

- Want cavity with large circulating power → on resonance
- Want Michelson to be on dark fringe (why? Thursday!)

Mirror positions and distance between objects can vary but interferometers only work (well) if positions and distances are well defined.

⇒ In the simulation, need to chose 'tuning' of mirrors carefully
In the experiment, control systems constantly measure and correct positions

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Degrees of freedom (DOF)

Michelson output : $a_6 = i a_1 e^{-i2k\bar{L}} \cos(2k\Delta L)$

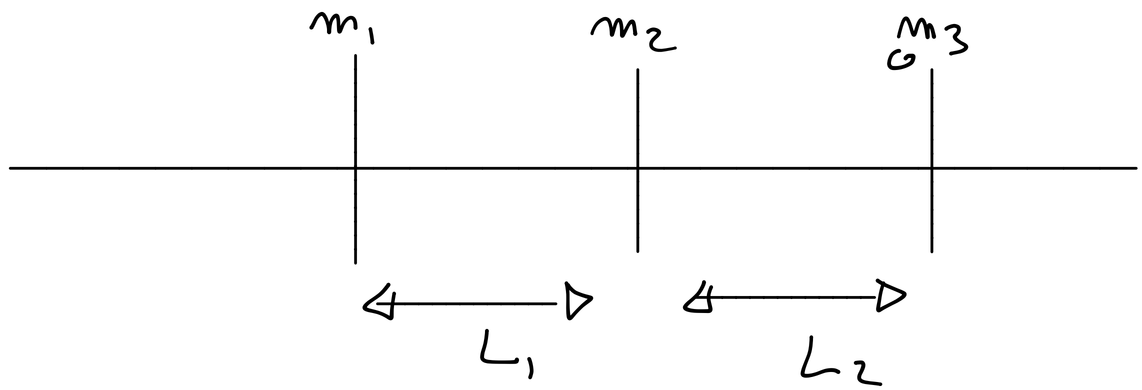
Chose to describe Michelson with $\bar{L}$, ΔL instead of L_1, L_2 because this highlight the behaviour of the instrument better.

$\bar{L}$ changes phase, ΔL changes amplitude, while L_1 or L_2 would change both.

These new variables are the degrees of freedom.

$\bar{L}$ common mode, ΔL differential mode

Important for all multi-optics systems, example coupled cavity:



To investigate move m_1 or m_3 but not m_2!

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Length signal and frequency noise

We want to measure length. L always appears as $\cos(2kL)$.

$k = \frac{\omega}{c} = \frac{2\pi f}{c}$, $\lambda = 10^{-6}$, $c = \lambda \cdot f$, $f = 3 \cdot 10^{14}$

$$f_0 = 3 \cdot 10^{14}$$

Assume $L = L_0 + \delta L$, $f = f_0 + \delta f$

L_0: constant; δL: signal; f_0: constant; δf: noise

Cavity (just look at $2kL$):

$$2kL = \frac{4\pi}{c} f \cdot L$$

$$= \frac{4\pi}{c}\left(f_0 L_0 + f_0 \delta L + \delta f L_0 + \underbrace{\delta f \delta L}_{\text{very small}}\right) \approx \frac{4\pi}{c}\left(\underbrace{f_0 \delta L}_{\text{signal}} + \underbrace{\delta f \cdot L_0}_{\text{noise}}\right)$$

$\frac{4\pi}{c} f_0 L_0 = N \cdot 2\pi$ on resonance

We need

$$f_0 \delta L > \delta f L_0 \Rightarrow \underbrace{\frac{\delta L}{L_0}}_{10^{-23}\,!!} > \frac{\delta f}{f_0}$$

10^{-15} for a very good laser!

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Same for Michelson

$$L = L_0 + \Delta L + \delta L \qquad \text{Power} \sim \cos(2k\Delta L)$$

constant

N·2π on dark fringe

very small

$$2k\Delta L = \frac{4\pi}{c}(f_0 + \delta f)(\Delta L + \delta L) = \frac{4\pi}{c}(f_0 \Delta L + f_0 \delta L + \delta f \Delta L + \delta f \delta L)$$

signal

noise

$$\approx \frac{4\pi}{c}(f_0 \delta L + \delta f \Delta L)$$

Now: $\frac{\delta f}{f_0} < \frac{\delta L}{\Delta L}$, $\Delta L \approx 10^{-2} m \Rightarrow \frac{\delta f}{f_0} < 10^{-17}$

6 orders of magnitude better than a cavity (and still difficult!)

Main reason for using a Michelson.

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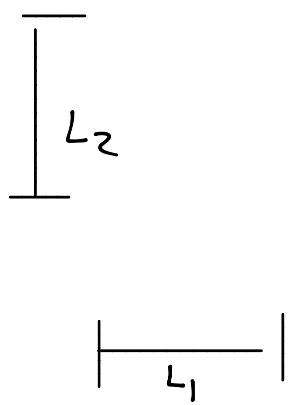
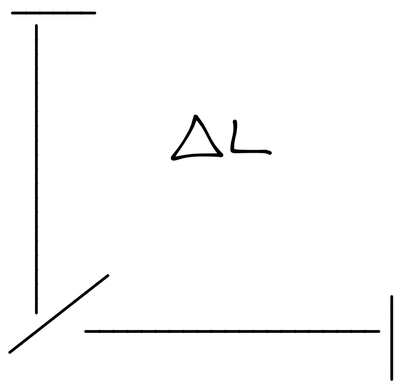
Why a Michelson?

- an interferometer scales signal by optical frequency $f_0 = 10^{14}$!
- measures GW signal 'twice'

- Michelson is many orders of magnitude (6!) less sensitive to freq. noise





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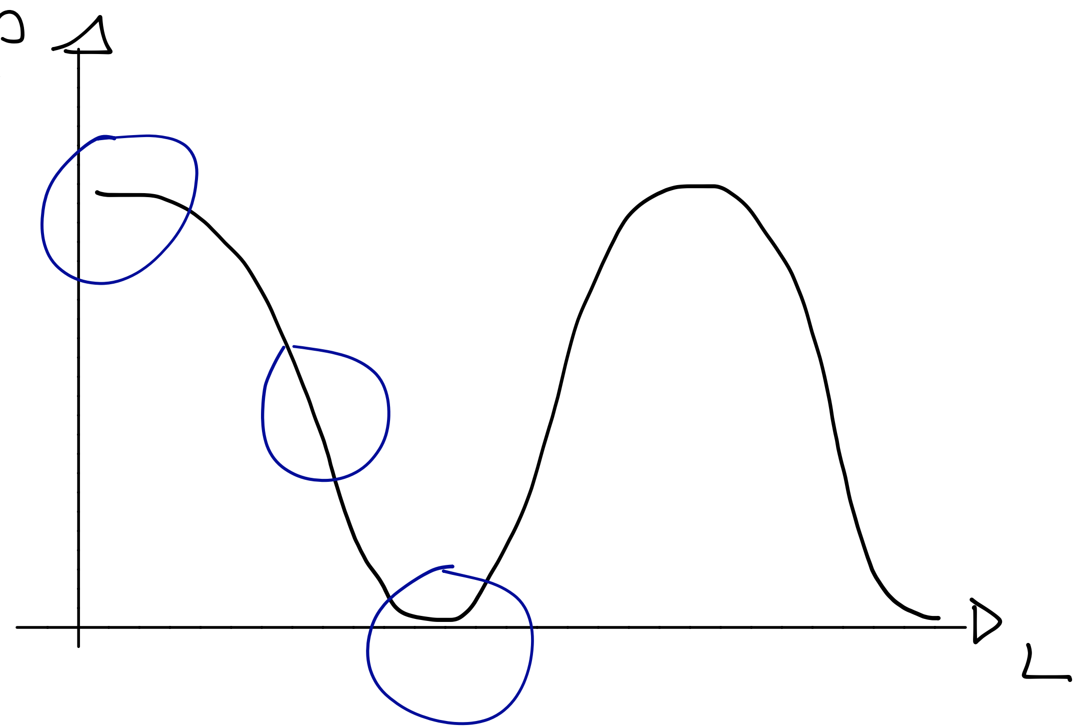
Signal gradient



$$\frac{dP}{d\Delta L} = -8Pk\sin(4k\Delta L)$$

Best 'sensitivity' at mid fringe.

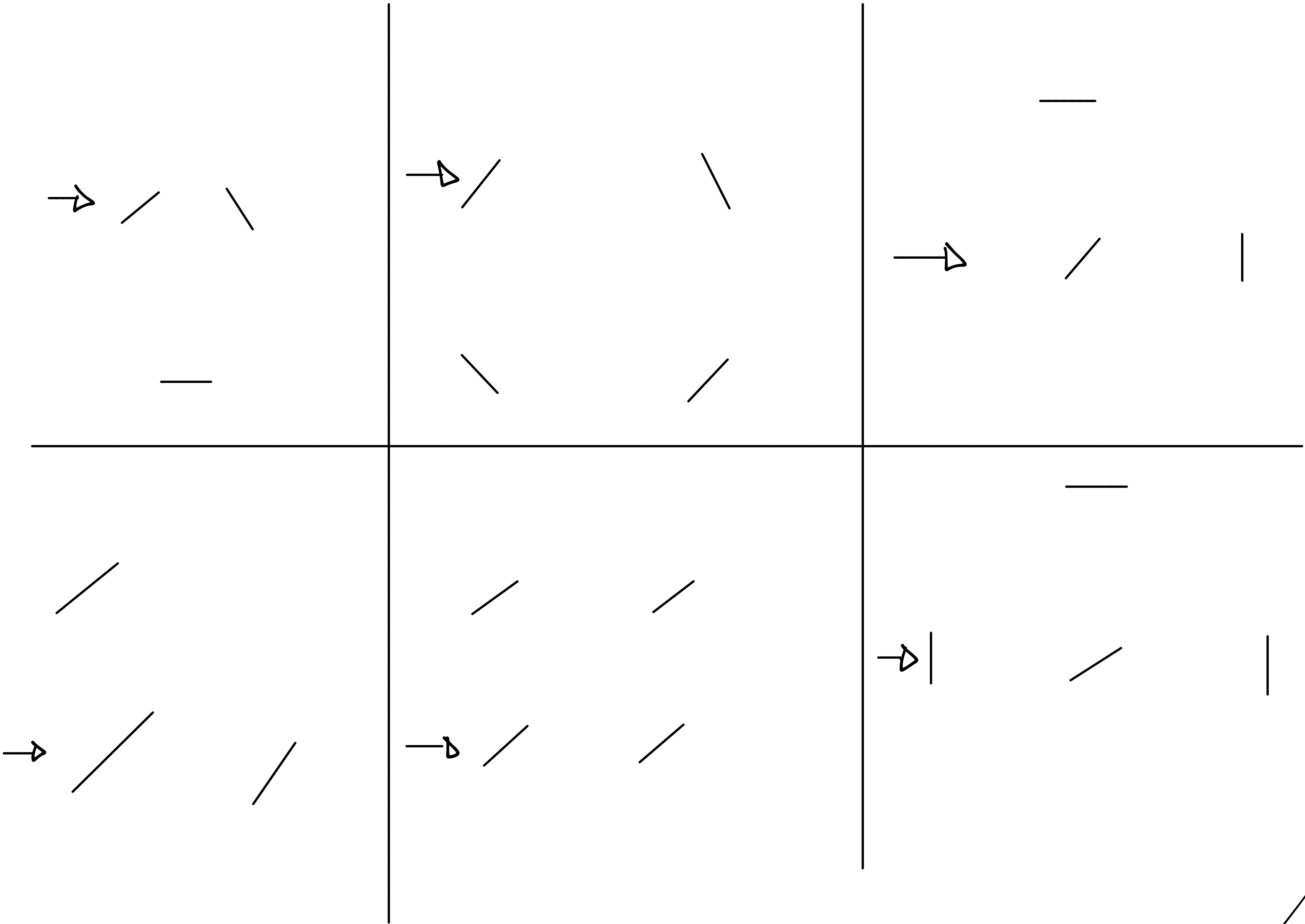
Why use dark fringe?

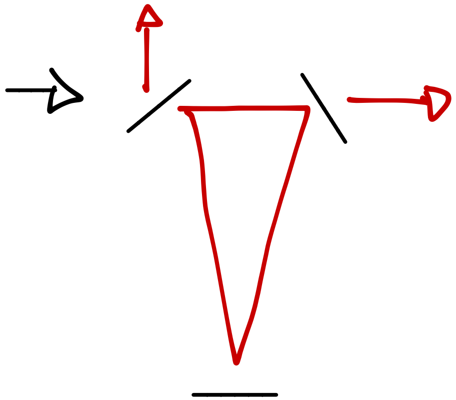
1) Slope misleading. Sensitivity really depends on signal vs. noise, e.g. mirror motion noise or laser freq. noise scale the same

2) Will see on Thursday that dark fringe will allow us to do a trick and gain laser power (for quantum noise reduction)

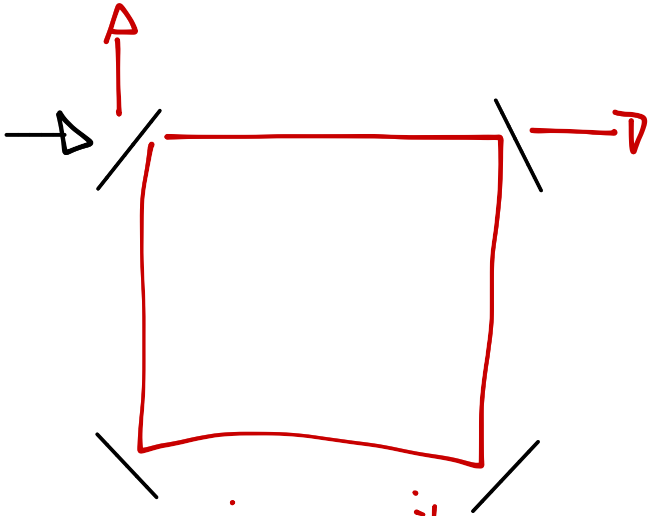
3) Generally good to do 'null experiments'.

At half fringe >50 W on photo diode.

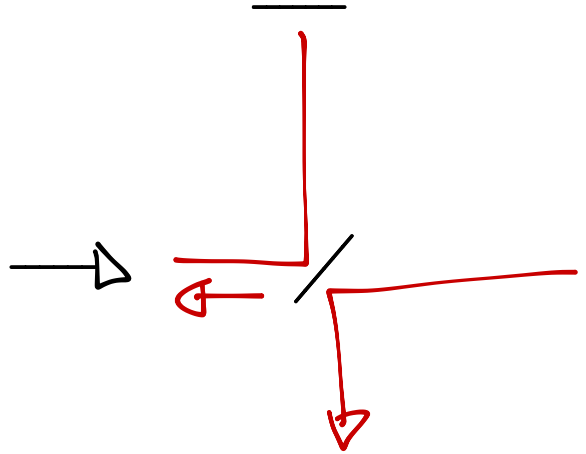




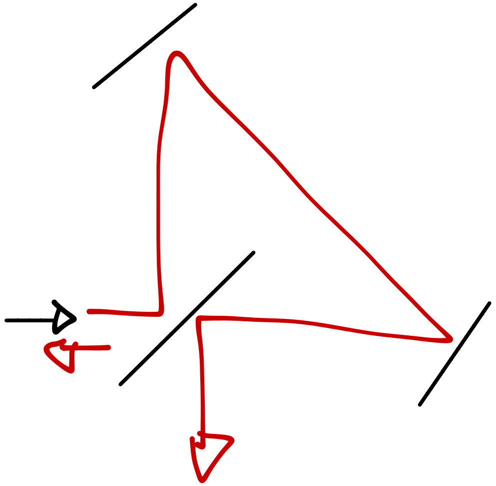
Triangular ring cavity



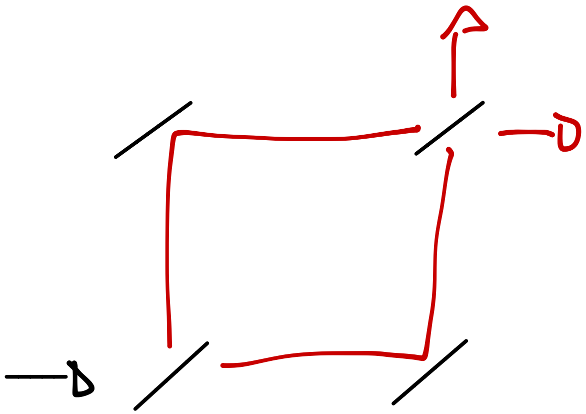
ring cavity



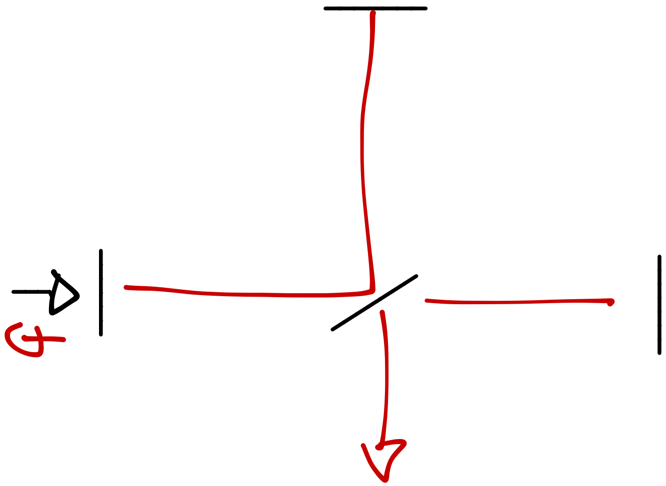
Michelson



Sagnac



Mach-Zehnder



Michelson with Power Recycling

Summary:

- equations for Michelson output
- Michelson has 2 degrees of freedom (common + differential)
- Michelson much less sensitive to laser frequency noise
- operate Michelson at 'dark fringe' (see Thursday)

Tomorrow:

- modulation of fields
- signals vs noise -> sensitivity
- transfer functions